

Conjunction Rules

$$\begin{array}{l}
 n_1 \mid P_1 \\
 \downarrow \\
 n_l \mid P_n \\
 \triangleright \mid P_1 \wedge P_2 \wedge \dots \wedge P_n \quad \wedge \mathbf{I}: n_1, \dots, n_l
 \end{array}$$

$$\begin{array}{l}
 n \mid P_1 \wedge P_2 \wedge \dots \wedge P_n \\
 \vdots \\
 \triangleright \mid P_i \quad \wedge \mathbf{E}: n
 \end{array}$$

Negation Rules

$$\begin{array}{l}
 m \mid \mid \begin{array}{l} P \\ \hline \vdots \\ \perp \end{array} \\
 n \mid \mid \perp \\
 \triangleright \mid \neg P \quad \neg \mathbf{I}: m-n
 \end{array}$$

$$\begin{array}{l}
 n \mid \neg \neg P \\
 \vdots \\
 \triangleright \mid P \quad \neg \mathbf{E}: n
 \end{array}$$

Bottom Rules

$$\begin{array}{l}
 m \mid P \\
 \vdots \\
 n \mid \neg P \\
 \vdots \\
 \triangleright \mid \perp \quad \perp \mathbf{I}: m, n
 \end{array}$$

$$\begin{array}{l}
 m \mid \perp \\
 \vdots \\
 \triangleright \mid S \quad \perp \mathbf{E}: m
 \end{array}$$

Biconditional rules

$$\begin{array}{l}
 m \mid \mid P \\
 \vdots \\
 n \mid \mid Q \\
 r \mid \mid Q \\
 \vdots \\
 s \mid \mid P \\
 \triangleright \mid P \leftrightarrow Q \quad \leftrightarrow \mathbf{I}: m-n, r-s
 \end{array}$$

$$\begin{array}{l}
 m \mid P \leftrightarrow Q \\
 \vdots \\
 n \mid P \\
 \vdots \\
 \triangleright \mid Q \quad \leftrightarrow \mathbf{E}: m, n \\
 m \mid P \leftrightarrow Q \\
 \vdots \\
 n \mid Q \\
 \vdots \\
 \triangleright \mid P \quad \leftrightarrow \mathbf{E}: m, n
 \end{array}$$

Disjunction Rules

$$\begin{array}{l}
 n \mid P_i \\
 \vdots \\
 \triangleright \mid P_1 \vee P_2 \vee \dots \vee P_n \quad \vee \mathbf{I}: n
 \end{array}$$

$$\begin{array}{l}
 n \mid P_1 \vee P_2 \vee \dots \vee P_n \\
 \vdots \\
 m_1 \mid \mid \begin{array}{l} P_1 \\ \hline \vdots \\ S \end{array} \\
 l_1 \mid \mid S \\
 \downarrow \\
 m_n \mid \mid \begin{array}{l} P_n \\ \hline \vdots \\ S \end{array} \\
 l_n \mid \mid S \\
 \vdots \\
 \triangleright \mid S \quad \vee \mathbf{E}: n, m_1-l_1, m_n-l_n
 \end{array}$$

Conditional rules

$$\begin{array}{l}
 m \mid \mid \begin{array}{l} P \\ \hline \vdots \\ Q \end{array} \\
 n \mid \mid Q \\
 \triangleright \mid P \rightarrow Q \quad \rightarrow \mathbf{I}: m-n
 \end{array}$$

$$\begin{array}{l}
 m \mid P \rightarrow Q \\
 \vdots \\
 n \mid P \\
 \vdots \\
 \triangleright \mid Q \quad \rightarrow \mathbf{E}: m-n
 \end{array}$$