Philosophy 57 Final March 19, 2004

This final is closed-book and closed-notes. There are 100 points total. Within each section, all questions are worth the same number of points. Don't forget to write your name!

1. **Syntax (4 points)** Indicate whether each string of symbols below is a *formula of first-order logic* by circling **Yes** if it is a formula, and **No** otherwise.

$\neg a = a$	**Yes**	**No**
$\neg x = x$	**Yes**	**No**
$x = \neg a$	**Yes**	**No**
$\forall x \mathsf{Cube}(y)$	**Yes**	**No**
$\forall x \forall x \exists x \mathsf{Cube}(x)$	**Yes**	**No**
x	**Yes**	**No**
$\forall x(x \vee \mathsf{Cube}(x))$	**Yes**	**No**
$\exists x(\mathsf{Tall}(x) \vee \mathsf{Short}(tim))$	**Yes**	**No**

2. **Syntax continued (3 points)** For each of the following formulas, circle **Yes** if the formula has any free variables, and **No** if does not have any free variables.

$\mathsf{Tet}(a)$	**Yes**	**No**
$\mathsf{Blue}(x)$	**Yes**	**No**
$\exists x \mathsf{Blue}(x) \wedge \mathsf{Green}(x)$	**Yes**	**No**
$\forall x \exists y \mathsf{P}(x, y)$	**Yes**	**No**
$\forall x \exists y \forall z \mathsf{P}(x, y)$	**Yes**	**No**
$\forall y(\mathsf{Mammal}(y) \rightarrow \mathsf{Animal}(y))$	**Yes**	**No**

3. **Semantic relations (14 points)** Indicate whether the sentences below are true or false by circling **T** or **F** for each.

(a)	$\forall x(x = x)$ is first-order necessary.	**T**	**F**
(b)	$\forall x \mathsf{SameSize}(x, x)$ is first-order necessary.	**T**	**F**
(c)	$\exists y \forall x P(x, y) \rightarrow \forall x \exists y P(x, y)$ is first-order necessary.	**T**	**F**
(d)	If $A \leftrightarrow B$ is a logical truth then A is first-order equivalent to B.	**T**	**F**
(e)	If $A \rightarrow B$ is a tautology then A first-order implies B.	**T**	**F**
(f)	All first-order necessary sentences are logically equivalent.	**T**	**F**
(g)	There is a first-order impossible sentence which is logically possible.	**T**	**F**

4. **Prenex form (6 points)** For each of the following sentences, find a sentence in prenex normal form that is (first-order) equivalent to the given sentence.

   (a) $\mathsf{Tall}(\mathsf{sally})$

   (b) $\forall x(\mathsf{Tall}(x) \lor \mathsf{Short}(x))$

   (c) $\neg\neg\neg\forall x \forall y \forall t \mathsf{Arrived}(x, y, t)$

(d) $\forall \mathsf{y} \neg (\forall \mathsf{x} \exists \mathsf{y} \mathsf{P}(\mathsf{x}, \mathsf{y}) \land \mathsf{S}(\mathsf{y}))$

(e) $\forall \mathsf{x} \forall \mathsf{y} (\forall \mathsf{z} \mathsf{R}(\mathsf{x}, \mathsf{y}, \mathsf{z}) \rightarrow \mathsf{Near}(\mathsf{x}, \mathsf{y}))$

(f) $\forall \mathsf{x} \mathsf{Blue}(\mathsf{x}) \leftrightarrow \exists \mathsf{x} \mathsf{Wood}(\mathsf{x})$

5. **Tautological form (3 points)** For each of the following arguments, indicate the tautological structure by writing up another argument with the same tautological structure that contains only 0-ary predicates and connectives. Then indicate whether the argument is tautologically valid. The first one is done for you.

   (a)
$$\begin{array}{|l} \forall x \mathsf{Cube}(x) \\ \hline \forall x \mathsf{Cube}(x) \end{array}$$

   Tautological structure:
$$\begin{array}{|l} \mathsf{A} \\ \hline \mathsf{A} \end{array}$$

   Argument is tautologically valid.

   (b)
$$\begin{array}{|l} \forall x(\mathsf{Man}(x) \rightarrow \mathsf{Mortal}(x)) \\ \mathsf{Man}(\mathsf{socrates}) \\ \hline \mathsf{Mortal}(\mathsf{socrates}) \end{array}$$

   (c)
$$\begin{array}{|l} \forall x(\mathsf{Small}(x) \land \mathsf{Red}(x)) \\ \hline \forall x(\mathsf{Red}(x) \land \mathsf{Small}(x)) \end{array}$$

(d)

$$\begin{array}{|l} \forall y \mathsf{Red}(y) \land (\forall x \exists y \mathsf{LooksLike}(x, y) \lor \neg \exists x \mathsf{Red}(x)) \\ \hline \forall x \mathsf{Red}(x) \end{array}$$

(e)

$$\begin{array}{|l} \forall y \mathsf{Mammal}(y) \rightarrow \forall y \mathsf{Animal}(y) \\ \neg \forall y \mathsf{Animal}(y) \\ \hline \neg \forall y \mathsf{Mammal}(y) \end{array}$$

(f)

$$\begin{array}{|l} \mathsf{Tall}(\mathsf{sally}) \\ \hline \exists x \mathsf{Tall}(x) \end{array}$$

6. **Translations (12 points)** Translate each of the following sentences into first-order logic. There may be more than one correct answer for some of them. Explain the meaning of any constant, function symbol, or predicate symbol that isn't self-evident.

   (a) Some computers are fast.

   (b) Only jedi have lightsabers.

   (c) There were no more than two books

   (d) All flashlights that need batteries eventually stop working.

(e) Every prince becomes king provided he has no older brothers.

(f) My vacation was better than any of the ones my friends took, except possibly for Tim.

7. **Translations continued (8 points)** Translate each of these English arguments into a *first-order valid argument*, composed of sentences of first-order logic. The first one is done for you.

   (a) Socrates is a man, and all men are mortal, so Socrates is mortal.

   $$\begin{array}{|l} \mathsf{Man(socrates)} \\ \forall \mathsf{x}(\mathsf{Man(x)} \rightarrow \mathsf{Mortal(x)}) \\ \hline \mathsf{Mortal(socrates)} \end{array}$$

   (b) Everyone loves someone. Bob doesn't love anyone but Sally. Thus, Bob loves Sally.

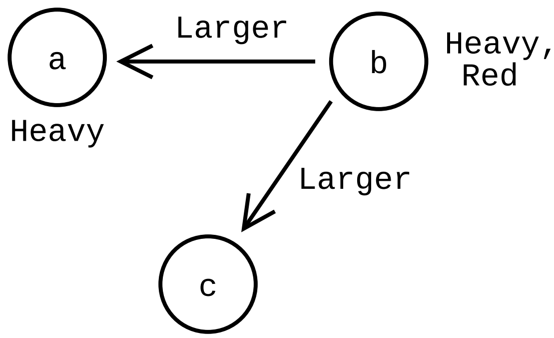
(c) Only the loudest person is selected. Connie is louder than Maria, so Maria wasn't selected.

(d) Commander Blaine owns only two watches, an IWC and an Omega. Any watch found at the scene was an Jaeger-LeCoultre (which of course is neither an IWC nor an Omega). Therefore no watches found at the scene were Blaine's.

(e) 2179 is prime. Therefore, it is not divisible by 17.

8. **Logic game (8 points)** Suppose you are the true player, and the world is as depicted below. For each sentence below, write what you should do, and what the new sentence would be. If you cannot determine what the new sentence will be (for instance because it is the false player's turn to choose), then just indicate that.

   The first one is done for you.



(a) $\mathsf{Heavy(c)} \lor \exists \mathsf{x}\mathsf{Heavy(x)}$

Choose $\exists \mathsf{x}\mathsf{Heavy(x)}$. New sentence is $\exists \mathsf{x}\mathsf{Heavy(x)}$.

(b) $\exists \mathsf{x}\mathsf{Heavy(x)}$

(c) $\forall \mathsf{x} \neg \exists \mathsf{y}(\neg \mathsf{x} = \mathsf{y} \rightarrow \mathsf{Larger(x, y)})$

(d) $\exists \mathsf{x}(\mathsf{Heavy(x)} \land \exists \mathsf{y}\mathsf{Larger(x, y)})$

(e) $\neg \exists \mathsf{x}(\mathsf{Heavy(x)} \leftrightarrow \forall \mathsf{x}\mathsf{Red(x)})$

9. **Counterexamples (12 points)** All of the following arguments are first-order invalid. For each, write or depict a first-order counterexample.

(a)

$$\begin{array}{|l} \forall x \mathsf{Small}(x) \\ \hline \mathsf{SameSize}(a, b) \end{array}$$

(b)

$$\begin{array}{|l} \forall x \exists y P(x, y) \\ \hline \exists y \forall x P(x, y) \end{array}$$

(c)

$$\begin{array}{|l}
\forall x \exists y \mathsf{Larger}(x, y) \\
\neg \exists x \mathsf{Larger}(x, x) \\
\hline
\exists x \exists y \exists z (\neg x = y \land \neg y = z \land \neg x = z)
\end{array}$$

(d)

$$\begin{array}{|l}
\forall x \exists y \exists z \exists w (R(x, y) \land R(y, z) \land R(z, w)) \\
\neg \exists x R(x, x) \\
\hline
\exists x \exists y (R(x, y) \land R(y, x))
\end{array}$$

10. **Proofs (20 points)** Write a formal proof for each of the following arguments. Don't use TautCon, AnaCon, or FOCon.

(a)

$$\begin{array}{|l} \forall x(\mathsf{Parisian}(x) \rightarrow \mathsf{French}(x)) \\ \exists x \mathsf{Parisian}(x) \\ \hline \exists x \mathsf{French}(x) \end{array}$$

(b)

$$\begin{array}{|l} \\ \hline \forall x \forall y(\mathsf{P}(y) \rightarrow \mathsf{P}(y)) \end{array}$$

(c)

$$\begin{array}{|l} \exists x \forall y \mathsf{Loves}(x, y) \\ \hline \forall y \exists x \mathsf{Loves}(x, y) \end{array}$$

(d)

$$\begin{array}{|l} \exists x \forall y R(x, y) \\ \forall x \exists y S(y, x) \\ \hline \exists x \exists y (R(x, y) \wedge S(y, x)) \end{array}$$

(e)
$$\begin{array}{|l} \exists x \forall y\, x = y \\ \hline \neg(\exists x P(x) \land \exists x \neg P(x)) \end{array}$$

11. **Axioms of Probability (6 points)** A probability measure P is a function from $\mathcal{L}$ to $\mathbb{R}$ (the real numbers) such that:

    1. $\mathrm{P}(S) \geq 0$ for all sentences S.

    2. If S is a logical truth, then $\mathrm{P}(S) = 1$.

    3. If A and B are mutually exclusive (i.e. $A \wedge B$ is logically impossible), then $\mathrm{P}(A \vee B) = \mathrm{P}(A) + \mathrm{P}(B)$.

    Prove the following two statements using only these axioms, and these additional facts:

    - $\mathrm{P}(\neg A) = 1 - \mathrm{P}(A)$
    - If A is logically equivalent to B, P(A) = P(B).
    - $\mathrm{P}(A \wedge B) \leq \mathrm{P}(A)$

    (a) $\mathrm{P}(A \vee B) \geq \mathrm{P}(A)$

    (b) If $\mathrm{P}(A) = 0$, then $\mathrm{P}(A \vee B) = \mathrm{P}(B)$.

12. **Conditional probability and independence (4 points)** The following two problems may be answered completely correctly with a single word and/or number. However, you may wish to show your work to ensure partial credit.

    (a) A fair coin is flipped 8 times. What is the probability that the final coin toss comes out heads, given that the first 7 were also heads?

    (b) Suppose

$$\begin{aligned} \mathrm{P}(A \wedge B) &= 0.18 \\ \mathrm{P}(A \wedge \neg B) &= 0.12 \\ \mathrm{P}(\neg A \wedge B) &= 0.42 \\ \mathrm{P}(\neg A \wedge \neg B) &= 0.28 \end{aligned}$$

    What is $\mathrm{P}(A)$? Are A and B independent?